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# Examples of Two-Wavenumber Spectra in Nonhomogeneous One-Dimensional Structures

## A Paper Presented at the Second Joint Meeting of the Acoustical Societies of America and Japan, Honolulu, Hawaii, 16 November 1988

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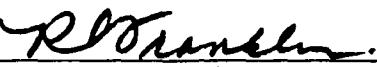
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PREFACE

This report was prepared under Project 638V11, Principal Investigator R.M. Kennedy (Code 3802). The work reported herein was performed as part of the Naval Underwater Systems Center Test and Evaluation Department Acoustic Range Initiative.

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19. ABSTRACT

Structures of interest to hydroacoustic research frequently have spatially varying properties which cause the field variable to be statistically nonhomogeneous. Two-wavenumber spectra of one-dimensional nonhomogeneous fields have been defined (W.A. Strawderman, NUSC Technical Document 7873, August 1987), but there exists little experience in interpreting these spectra. This paper presents a first look at the two-wavenumber spectra of various theoretical and measured examples of nonhomogeneous fields. The examples were chosen to form a progression of two-wavenumber spectra from simple canonical forms of exponentially damped traveling waves to more complex measured nonhomogeneous fields on axisymmetric periodic structures. The desired two-wavenumber spectra are obtained from calculated or measured cross-spectral density matrices by use of an efficient algorithm. A comparison of the spectra of both the canonical and measured fields display a common structure. One- and two-wavenumber spectra of measured data are compared to illustrate the potential simplification and interpretive role of two-wavenumber spectra.

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This document is a transcript of the presentation given by Dr. Kennedy to the Second Joint Meeting of the Acoustical Society of America and the Acoustical Society of Japan on November 16, 1988.



A-1

## INTRODUCTION

This presentation will describe our initial experience with the measurement and interpretation of the two-wavevector-frequency spectra of nonhomogeneous but stationary fields. Let us begin with the definitions appropriate to the nonhomogeneous stationary case.

### SLIDE 1

As shown at the top of this slide, a space-varying spectrum is defined as the Fourier transform of the correlation of a nonhomogeneous, stationary field variable that is a function of an absolute spatial location,  $\underline{x}$ , of one observation of the field variable, the spatial separation,  $\underline{\xi}$ , between observations, and the time delay,  $T$ , between observations.

Recall, from Dr. Strawderman's earlier talk, that the two-wavevector-frequency spectrum of a nonhomogeneous, stationary field variable is obtained by taking a Fourier transform of the space-varying spectrum, as shown at the bottom of the slide. The wavevector,  $\underline{\mu}$ , is the Fourier conjugate variable of the absolute spatial vector,  $\underline{x}$ ; the wavevector,  $\underline{k}$ , is the conjugate variable of the spatial separation vector,  $\underline{\xi}$ ; and the frequency,  $\omega$ , is the conjugate variable of the time delay,  $T$ . Note that the number of the wavevectors and their dimensions must exactly match the number of spatial variables and their dimensions that are present in the correlation field. Spatial spectra resulting from nonhomogeneous fields must necessarily have a higher dimensionality than its homogeneous counterpart because the correlation field is a function of both a location variable and a separation variable.

**Definition of the Two-Wavevector-Frequency Spectra  
(Stationary, Nonhomogeneous Field)**

**Space-Varying Spectrum ( $K_p$ )**

$$K_p(\underline{x}, \underline{k}, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underline{x}, \underline{\xi}, \tau) \exp[-i(\underline{k} \cdot \underline{\xi} + \omega \tau)] d\underline{\xi} d\tau$$

**Where Space-Time Correlation Is**

$$Q_{pp}(\underline{x}, \underline{\xi}, \tau) = E \{ p(\underline{x}, t) p(\underline{x} + \underline{\xi}, t + \tau) \}$$

**Two-Wavevector Spectrum ( $K_p$ )**

$$K_p(\underline{\mu}, \underline{k}, \omega) = \int_{-\infty}^{\infty} K_p(\underline{x}, \underline{k}, \omega) \exp[-i \underline{\mu} \cdot \underline{x}] dx$$

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Experimentally this increase in the dimensionality of the problem represents a significant complication, for two reasons. First, the process of viewing wavevector-frequency spectra is difficult to begin with because the surface, in general, exceeds three dimensions for homogeneous fields. In the case of a stationary nonhomogeneous field one is faced with the task of examining an eight-dimensional figure unless the physics of the problem reduce the number of nonhomogeneous directions. The second difficulty is the lack of experience with the wavevector spectra associated with the length variable characterizing the nonhomogeneous nature of the problem. Thus, even after coming to grips with the analytical and numerical problems of the spectral estimation, we are faced with such data analysis problems as "what does it look like?" and "is it useful?" or more exactly "was it worth doing?" In this presentation we examine these questions for an inherently low dimensional structure; that is, a linear hydrophone array in near axial flow.

SLIDE 2

The previous slide illustrated the conceptual form of the desired quantity. However, other forms are available and are important when developing spectral estimation algorithms, as will be shown. In this slide we show a computational form as well as the preferred conceptual form. The computational form is seen to be nested Fourier transforms of the fundamental spatial variables of the problem. The second equation gives the computational form in terms of the experimentally important quantity: cross-spectral density. However, the resulting wavevector does not maintain its familiar relation to the correlation separation variable and thus eliminates decades of experience. The preferred conceptual form maintains this relation; thus, we use the computational form initially and then by a transformation complete the calculation with the conceptual form.

Forms of Two-Wavevector-Frequency Spectra  
(Stationary, Nonhomogeneous Field)

Computational Form

$$\begin{aligned}
 K'_p(\nu, \gamma, \omega) &= \iint_{-\infty}^{\infty} Q_{pp}(\underline{x}_1, \underline{x}_2, \tau) \exp[-i(\nu \cdot \underline{x}_1 + \gamma \cdot \underline{x}_2 + \omega \tau)] d\underline{x}_1 d\underline{x}_2 d\tau \\
 &= \iint_{-\infty}^{\infty} S_{pp}(\underline{x}_1, \underline{x}_2, \omega) \exp[-i(\nu \cdot \underline{x}_1 + \gamma \cdot \underline{x}_2)] d\underline{x}_1 d\underline{x}_2
 \end{aligned}$$

Where  $S_{pp}(\underline{x}_1, \underline{x}_2, \omega)$  is a Cross-Spectra Density

Conceptual Form

$$K_p(\mu, \kappa, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underline{x}_1, \underline{x}_1 + \underline{\xi}, \tau) \exp[-i(\mu \cdot \underline{x}_1 + \kappa \cdot \underline{\xi} + \omega \tau)] d\underline{x}_1 d\underline{\xi} d\tau$$

SLIDE 3

In much of our experimental work our signal-processing equipment is designed to efficiently calculate the matrix of cross-spectral densities associated with an experiment containing  $n$  transducers. Thus the cross-spectral density (CSD) matrix is a good starting point for a two-wavevector calculation. This slide shows the computational form of the two-wavevector-frequency spectra in terms of the CSD matrix components for a one-dimensional structure.

## Calculation of Two-Wavenumber Spectra From a CSD Matrix

$$\begin{aligned}
 X(0) &= -\frac{N-1}{2} L \\
 X(n) &= \left[ n - \frac{N-1}{2} \right] L \\
 X(N-1) &= \frac{N-1}{2} L
 \end{aligned}$$

$$\hat{K}_{p'}(\nu, \gamma, \omega) = L^2 \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} w_n w_m C_{nm}(\omega) \exp(-i[n\nu+m\gamma]L) \exp(i \frac{N-1}{2} L [\nu+\gamma])$$

where

$C_{nm}(\omega)$  =  $n$ th,  $m$ th CSD matrix element at frequency  $\omega$

$w_n$  = spatial weights

$\hat{K}_{p'}$  estimates  $K_{p'}$

SLIDE 4

If we solve the previous equation at the unique set of wavenumbers associated with the fast Fourier transfer algorithm, then the resulting two-wavevector-frequency spectral estimation algorithm is seen to be a pair of nested FFTs. A final transformation yields the preferred form.

Evaluate  $\hat{\kappa}'_p$  at the FFT Wavenumbers

$$\begin{aligned}\nu_p &= \frac{2\pi}{NL} p \quad \text{for} \quad 0 \leq \nu_p \leq \frac{\pi}{L} \\ \nu_p &= \frac{2\pi}{NL} p - \frac{2\pi}{L} \quad \text{for} \quad -\frac{\pi}{L} < \nu_p < 0 \\ \gamma_q &= \frac{2\pi}{NL} q \quad \text{for} \quad 0 \leq \gamma_q \leq \frac{\pi}{L} \\ \gamma_q &= \frac{2\pi}{NL} q - \frac{2\pi}{L} \quad \text{for} \quad -\frac{\pi}{L} < \gamma_q < 0\end{aligned}$$

Substitutions of these wavenumbers yields

$$\begin{aligned}\hat{\kappa}'_p(\nu_p, \gamma_q, \omega) &= L^2 \exp\left[i\pi\left(\frac{N-1}{N}\right)(p+q)\right] \\ &\times \sum_{n=0}^{N-1} w_n \exp\left[-i\frac{2\pi}{N} np\right] \sum_{m=0}^{N-1} w_m C_{nm}(\omega) \exp\left[-i\frac{2\pi}{N} mq\right]\end{aligned}$$

Transformation to preferred form

$$\hat{\kappa}_p(\mu, k) = \hat{\kappa}'_p(\nu, \gamma)$$

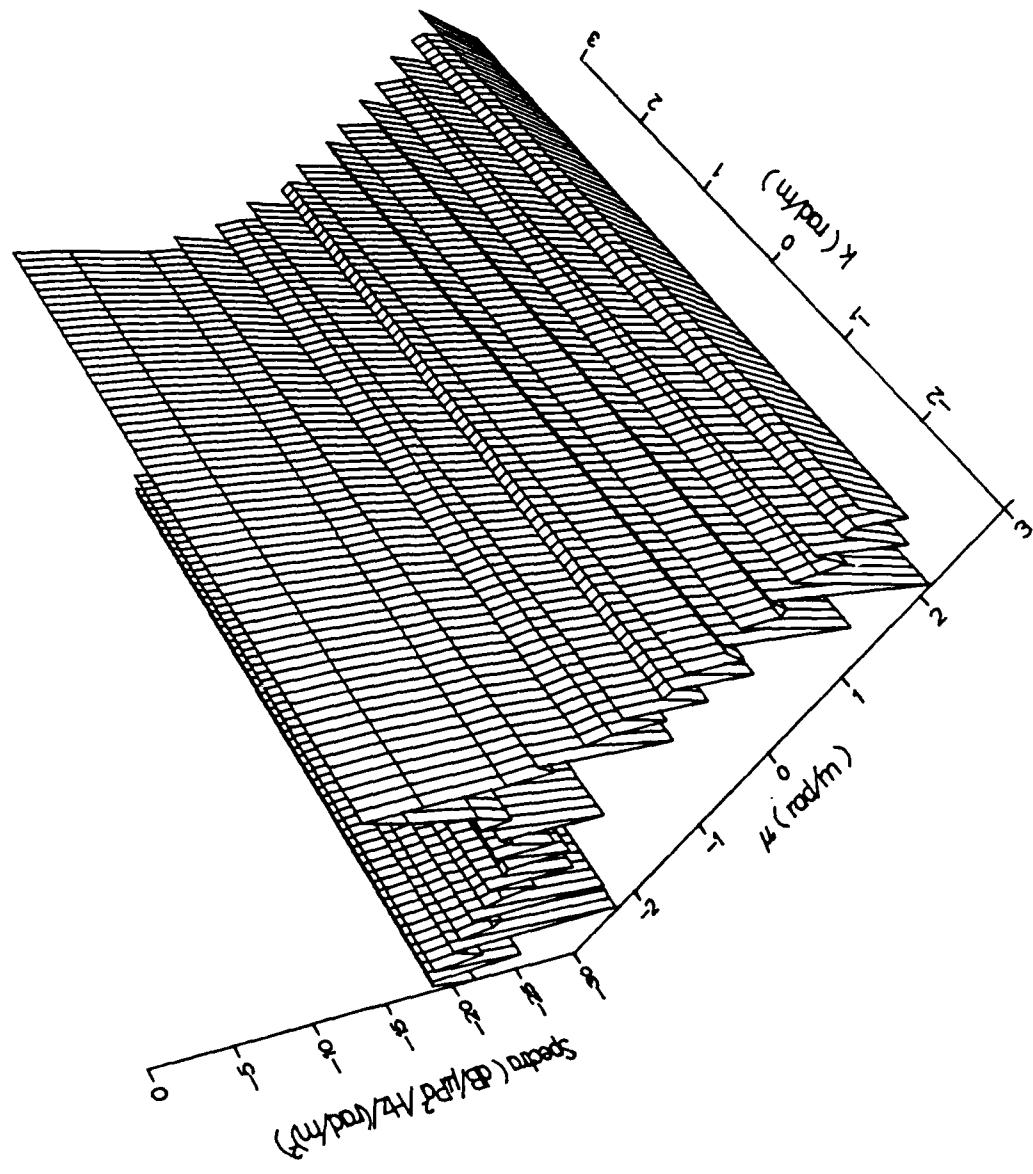
where

$$k = \gamma, \quad \mu = \nu + \gamma$$

## SLIDE 5

In the remainder of this presentation we will show examples of one- and two-wavenumber-frequency spectra. First, canonical forms of correlation fields will be used to illustrate fundamental "footprints" that will also be observed in the measured data. This step-by-step approach was necessary because of lack of experience with the subject density function. In this slide we show the simplest field possible; that is, a homogeneous one for which the CSD matrix is just the identity matrix. The  $k$  wavenumber is associated with the separation variable, and the  $\mu$  wavenumber is associated with the location variable. Because the field is homogeneous, there is no energy in the  $\mu$  dimension; and because the field is isotropic, the energy in the  $k$  dimension is wavenumber white. Note that at all nonzero values of  $\mu$  the spectral values result from "side lobe leakage" in the estimation algorithm.

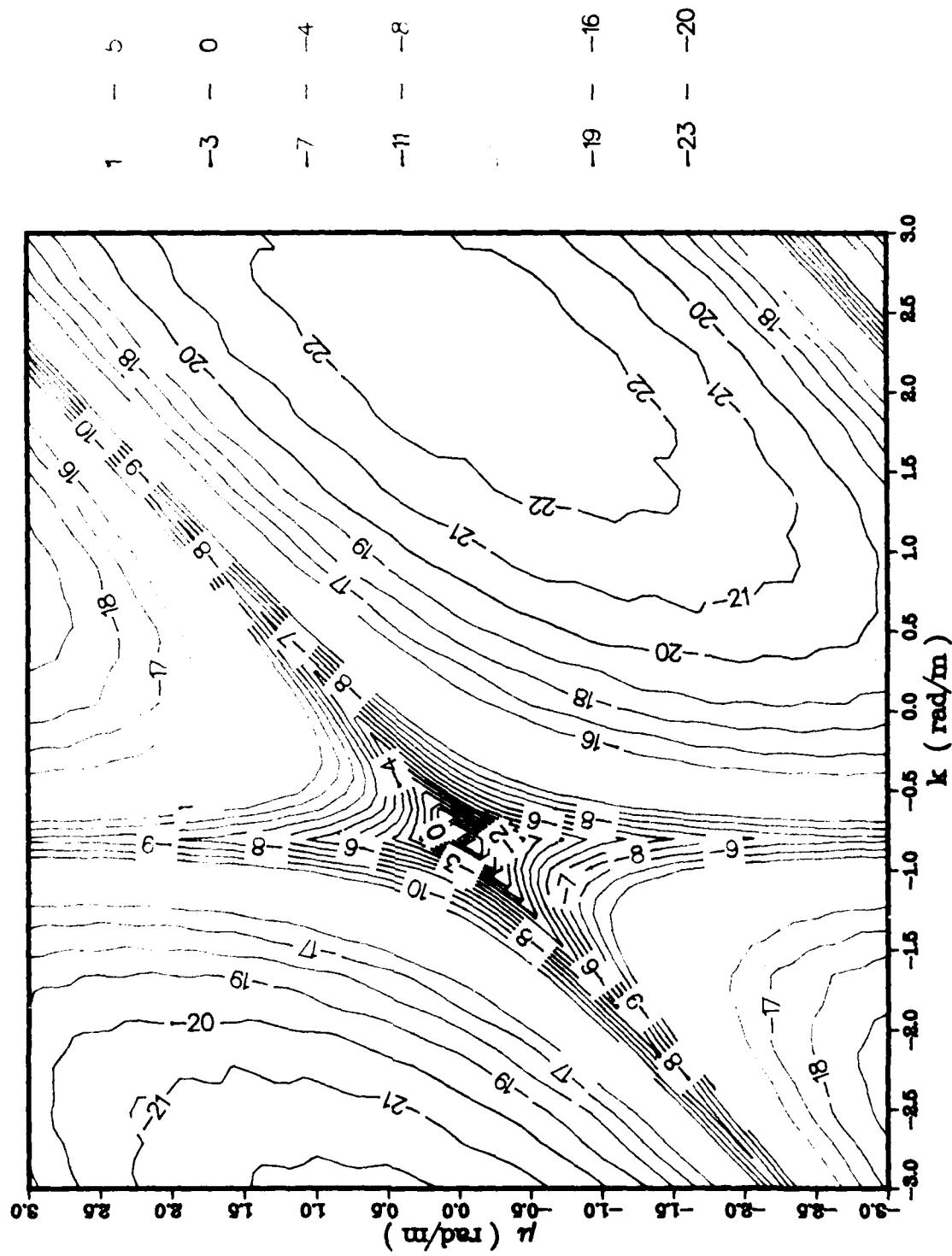
## Homogeneous Isotropic Example



## SLIDE 6

In this slide we show a fundamental pattern. The figure is a contour plot of the three-dimensional space associated with the calculated two-wavenumber-frequency spectra. The contours are constant values of a pressure density function with the dimensions  $\mu\text{Pa}^2\text{Hz}(\text{rad}/\text{m})^2$ . The spectrum shown here results from a unidirectional traveling wave decaying exponentially in space. The peak in the  $k$  dimension is the familiar propagation wavenumber. Because the field is now nonhomogeneous, there is a distinctive distribution of energy in the  $\mu$  dimension.

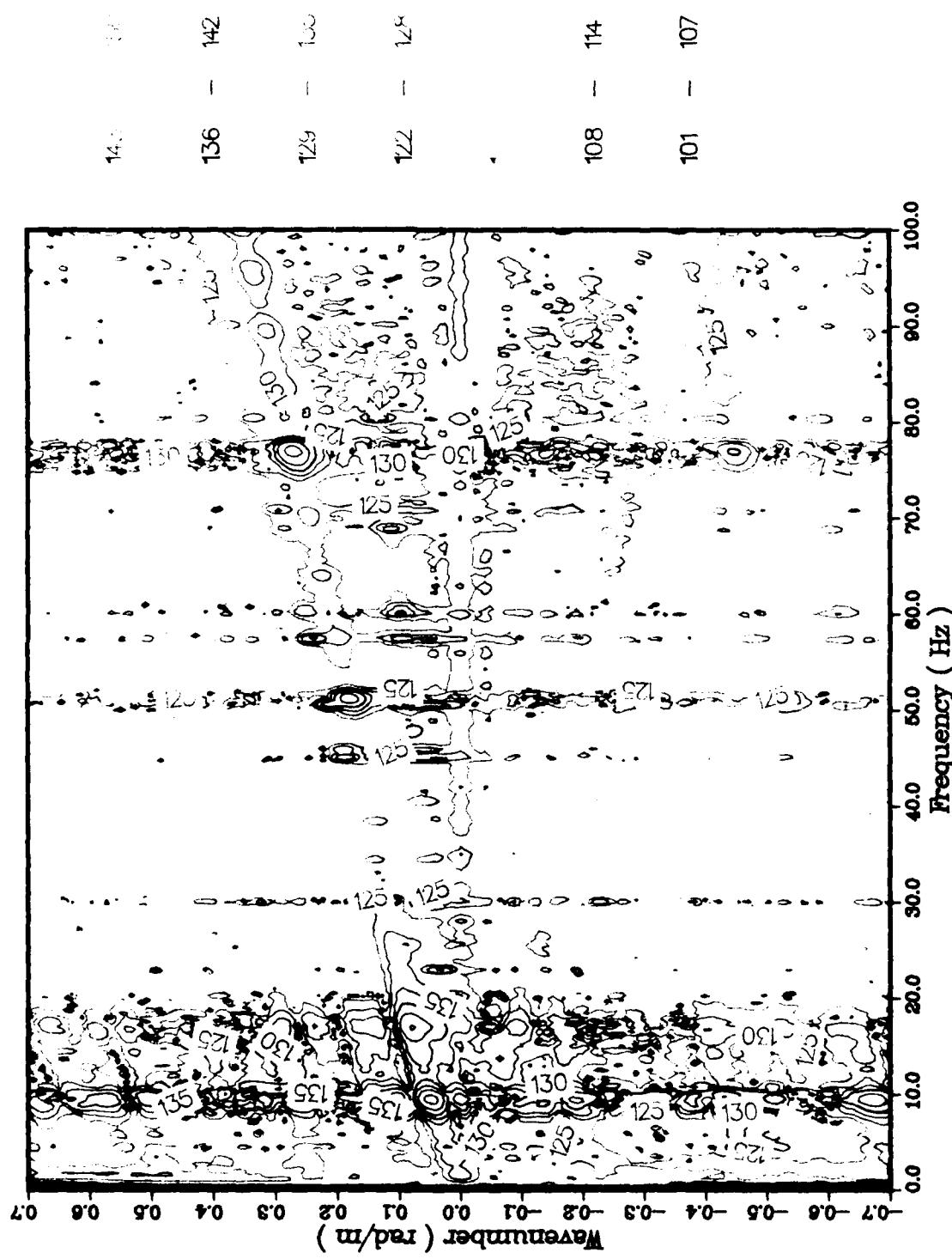
## Unidirectional Exponentially Damped Example



## SLIDE 7

Before showing measured two-wavenumber spectra, it is useful to examine the more familiar one-wavenumber-frequency spectral density of the measurement. This is equivalent to the spaced average spectra discussed by Dr. Strawderman. Recall that the structure is a linear array of equally spaced hydrophones in axial flow. The numerical details of the experiment are not important to this presentation, nor does time allow a detailed description. To orient you to the figure, note the obvious "sonic" cone that separates the acoustic from nonacoustic wavenumbers. Significant amounts of energy are present in both domains. Next please note the narrow band of energy that extends across all wavenumbers at 9.5 Hz. This energy results from a supersonic longitudinal vibration in the structure. The structure consists of a chain of long sections that creates a periodic mechanical discontinuity at the juncture of the sections. The measured pressure field contains "hot spots" at each of the junctures, creating a distinctly nonhomogeneous pressure field. This periodic modulation of the amplitude of the pressure field manifests itself in the one-wavenumber spectra as periodic "sidebands" of energy with a separation proportional to the reciprocal of the lengths of the sections. The wavenumber structure is quite complicated, and only rather obvious qualitative information can be realized from this figure.

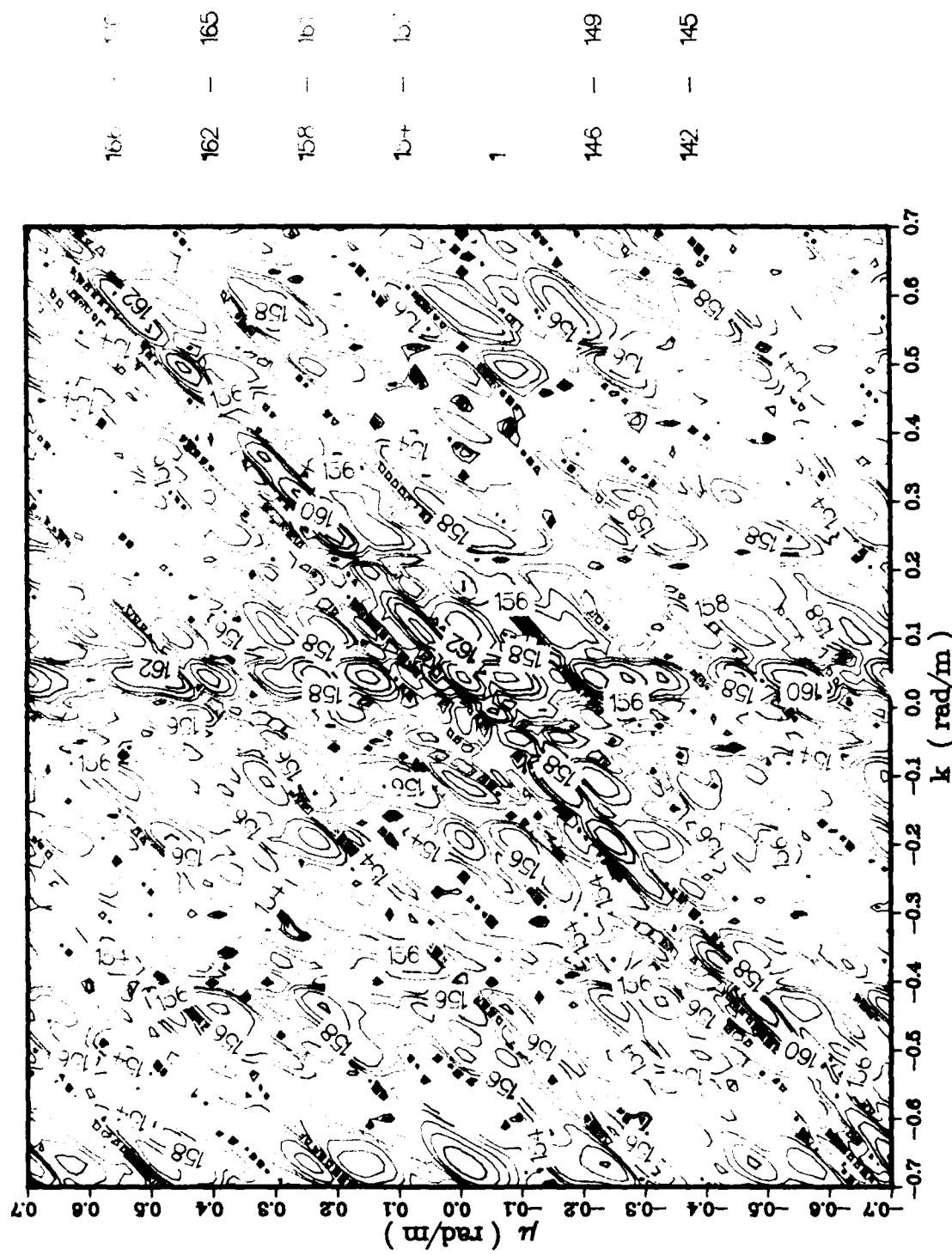
## Measured One Wavenumber – Frequency Spectra



## SLIDE 8

This slide illustrates the two-wavenumber-frequency spectra of the measurement at a single frequency; that is, 9.5 Hz. The horizontal axis is the homogeneous wavenumber  $k$ , and the vertical axis is the nonhomogeneous wavenumber  $\mu$ . One immediately sees the "footprint" of an exponentially decaying longitudinal wave. Quantitative analysis of the  $\mu$  dependence of the energy is a measure of the damping factor of the wave. The velocity of the wave is obtained from the peak in the pattern at  $\mu$  equal to zero. Note also the apparent lack of a homogeneous component to the field as evident from the lack of a ridge in  $k$  at zero  $\mu$ .

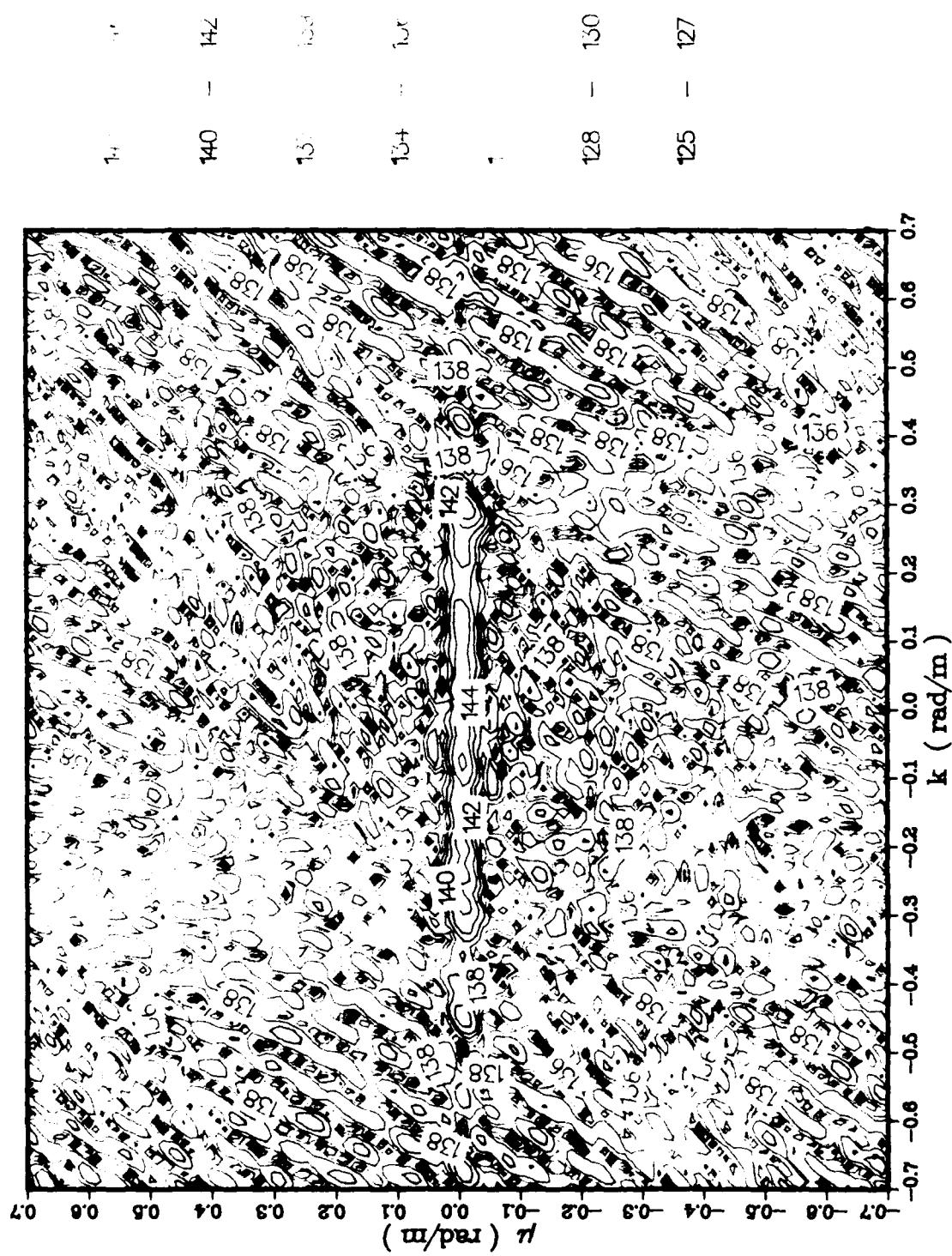
## Measured Nonhomogeneous Two Wavenumber – Frequency Spectra



SLIDE 9

In this slide we show the two-wavenumber spectra of the data at a frequency of 79 Hz. This frequency was chosen because it contained little energy from the previously examined vibrational wave and also because it contained significant sonic energy. Here we see clear evidence of homogeneous acoustic energy and some homogeneous nonacoustic waves. However, the general nonacoustic energy is, strangely, uniformly nonhomogeneous.

Measured Acoustic Ambient  
Two Wavenumber – Frequency Spectra



SLIDE 10

In summary, we have shown examples of two-wavenumber spectra calculated from both theoretical and measured cross-spectral density matrices. Canonical forms of common fields were used to establish identifiable patterns that were then observed in the data. In particular, the approach appears to be useful in the quantitative identification of nonhomogeneous, exponentially damped waves.

## SUMMARY

- Two-Wavenumber-Frequency Spectra Were Calculated and Displayed From Theoretical and Measured Cross-Spectral Density Matrices.
- Canonical Forms of Homogeneous and Nonhomogeneous Pressure Fields Were Used to Establish Identifiable Multidimensional Patterns To Be Compared With Measured Data.
- Comparisons Between the Canonical Forms and Measurements of Towed Hydrophone Lines Illustrate That Nonhomogeneous Exponentially Damped Structural Waves Are Clearly Identified by Two-Wavenumber-Frequency Spectra.

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